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K - 12 Mathematics

Mathematics Framework

Calgary Board of Education

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Notes |

1| The Mathematics Framework is intended for CBE employees, and as such, may link to resource materials available on Insite, CBE’s staff-facing intranet.
2| The use of the terms “student(s)” within the Mathematics Framework encompasses all learners within CBE, including Kindergarten children. The terms “student(s)” and “learner(s)” are used synonymously.

Mathematics Framework

Outcome

“Each and every [student] should develop deep mathematical understanding as confident and capable learners; understand and critique the world through mathematics; and experience the wonder, joy, and beauty of mathematics” (NCTM, 2020a, p. 9).

The learning of school mathematics encompasses multiple purposes: inspire students, foster positive mathematical identities, and empower students to engage in society with mathematical skills and understandings (NCTM, 2018; NCTM, 2020a; NCTM, 2020b). These purposes lead to learners having a deep understanding of mathematics, using mathematics as a lens through which to view the world, and “[experiencing] the wonder, joy, and beauty of mathematics” (NCTM, 2020a, p. 9). Having broader purposes of learning mathematics allows for all students to engage with mathematics in personal and culturally relevant ways and experience success as “knowers, doers, and sense-makers of mathematics” (NCTM, 2020b, p. 18).

There is a growing consensus among mathematicians, scholars, and educators about *what* mathematics should be taught as well as *how* it should be taught (NCTM, 2014). In terms of *what* is taught, the Alberta Mathematics Curriculum or Program of Studies outlines the mathematics content for each grade or course. Learning curriculum content includes learning and using mathematical processes. Mathematical processes are also developed while learning content (NCTM, 2000).

In the implementation of curriculum through instructional design, the *how* is equally important. The Mathematics Framework outlines the *how* of teaching and learning mathematics in Calgary Board of Education (CBE). The mathematics teaching and assessment practices, mathematics environment, and mathematics equity practices guide our work in transforming curriculum into mathematics experiences that support success for all learners.

By developing professional capacity through this framework, we work towards achieving the key outcome from the CBE Education Plan | 2021-2024, “Students achieve excellence in mathematics” (p. 11). Achieving excellence is focused on supporting every student to realize their full potential, and prioritizing professional learning and well-being of employees.

To support the key outcome of achieving excellence in mathematics, the CBE’s requirement for instructional time in mathematics represents an increase of 25% over Alberta Education’s recommended time allotment for mathematics.

This table shows Alberta Education’s recommended yearly hours, and CBE’s required times per day as averaged over the year:

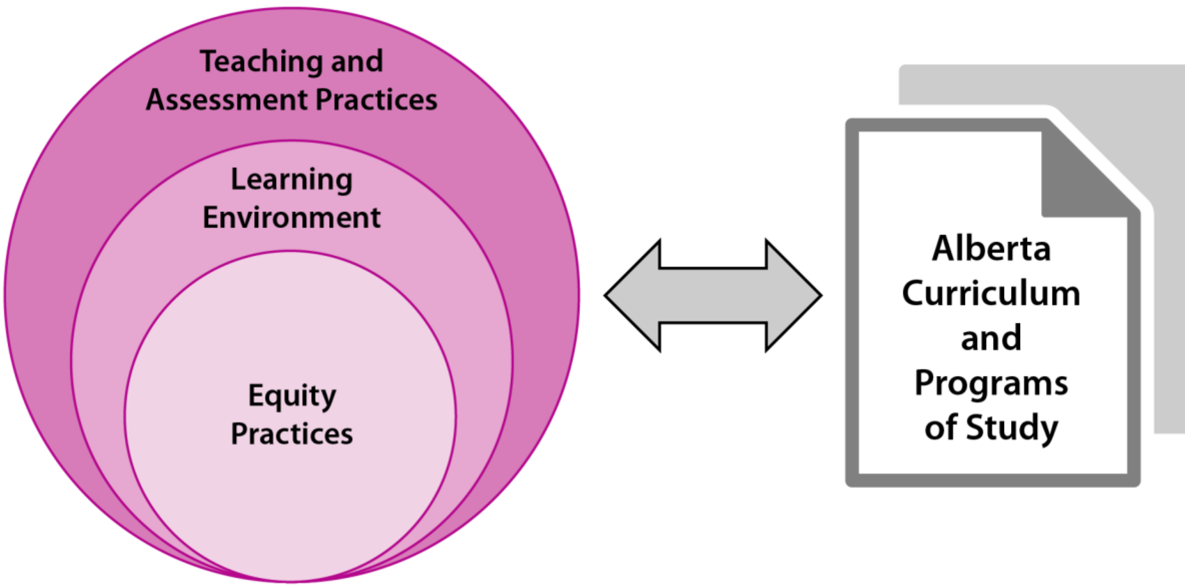
	Alberta Education	CBE
Grades 1 to 6	*15% of 950 hours. This results in 142.5 hours. Based on 180 days of instruction, this equates to 47.5 minutes.	1 hour
Grades 7 to 9	*100 hours out of 950 hours. Based on 180 days of instruction, this equates to 33.3 minutes.	42 minutes

*as stated in the Guide to Education

Essential Elements

"The teaching practices within the [Mathematics Framework] are a coherent and connected set of practices that when implemented together, create a classroom learning environment supportive of equitable teaching practices" (Berry III, R. Q., 2019, May).

The essential elements of the CBE Mathematics Framework are mathematics teaching and assessment practices, the mathematics learning environment, and mathematics equity practices. These elements, along with considerations for implementation, are in line with enduring research that focuses on best practices in mathematics education.



Mathematics Processes

In learning to think and act like a mathematician, students will experience and engage with mathematics through a set of processes integral to the discipline. Each process is threaded through all stages of planning, task design, teaching and learning, and assessment. Mastery of learning outcomes can only be demonstrated through a combination of curricular proficiency and use of these mathematical processes. Fundamental mathematical processes, which are integral to daily instruction and planning, can be categorized as problem solving, reasoning and proof, communication, connections and representation (NCTM, 2000). Similar processes are also embedded within the Alberta 4-12 Mathematics Programs of Study.

<table>
<tr><th>Mathematics Processes</th></tr>
<tr><td>Problem Solving
Mathematics instruction supports students to:

Build new mathematical knowledge through problem solving
Solve problems that arise in mathematics and in other contexts
Apply and adapt a variety of appropriate strategies to solve problems
Monitor and reflect on the process of mathematical problem solving
</td></tr>
<tr><td>Reasoning and Proof
Mathematics instruction supports students to:

Recognize reasoning and proof as fundamental aspects of mathematics
Make and investigate mathematical conjectures
Develop and evaluate mathematical arguments and proofs
Select and use various types of reasoning and methods of proof
</td></tr>
<tr><td>Communication
Mathematics instruction supports students to:

Organize and consolidate their mathematical thinking through communication
Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
Analyze and evaluate the mathematical thinking and strategies of others
Use the language of mathematics to express mathematical ideas precisely.
</td></tr>
<tr><td>Connections
Mathematics instruction supports students to:

Recognize and use connections among mathematical ideas
Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
Recognize and apply mathematics in contexts outside of mathematics
</td></tr>
<tr><td>Representation
Mathematics instruction supports students to:

Create and use representations to organize, record, and communicate mathematical ideas
Select, apply, and translate among mathematical representations to solve problems
Use representations to model and interpret physical, social, and mathematical phenomena
</td></tr>
</table>

(NCTM, 2000, p. 402 – sentence stems altered)

Mathematics Equity Practices

"Access and equity in mathematics at the school and classroom levels rest on beliefs and practices that empower all students to participate meaningfully in learning mathematics and to achieve outcomes in mathematics that are not predicted by or correlated with student characteristics" (NCTM, 2014, p. 60).

Considerations for implementation:

The five equity-based practices outlined in *Taking Action: Implementing Effective Mathematics Teaching Practices* (Boston et al., 2017, p. 6; Huinker & Bill, 2017, p. 6; Smith et al., 2017, p. 6) are:

I. **Go deep with mathematics.** Develop students' conceptual understanding, procedural fluency, and problem solving and reasoning.
II. **Leverage multiple mathematical competencies.** Use students' different mathematical strengths as a resource for learning.
III. **Affirm mathematics learners' identities.** Promote student participation and value different ways of contributing.
IV. **Challenge spaces of marginality.** Embrace student competencies, value multiple mathematical contributions, and position students as sources of expertise.
V. **Draw on multiple resources of knowledge (mathematics, language, culture, family).** Tap students' knowledge and experiences as resources for mathematics learning.

Mathematics Environment

For instructional and assessment practices to be successful, the mathematics environment must be carefully considered. Particular attention needs to be paid to beliefs about mathematics, the mathematical identities of teachers and students, and classroom routines.

“Teachers proactively create learning environments and pedagogy accessible to all students and encourage students to become more autonomous learners by embedding choices and removing learning barriers in their instruction” (Eichhorn et al., 2019, p. 264).

Beliefs

In CBE, we believe mathematics is:

I. **A human construct.** Each individual has natural sense-making abilities and learns mathematics through daily experiences.
II. **A historical and cultural endeavor**. Mathematics has deep roots in all cultures and reflects people’s needs to measure and communicate about time, quantity, distance, shape, and change.
III. **Creative and beautiful.** Mathematical creativity lies in the diverse approaches, representations, and ways of thinking about mathematics.
IV. **Both a discipline and an interdisciplinary tool**. Ideas in all disciplines can be modelled and communicated mathematically.

Mathematics classrooms are places where students believe (Boaler, n.d.):

- Everyone can do well in [math].
- Mathematics problems can be solved with many different insights and methods.
- Mistakes are valuable; they encourage brain growth and learning.
- Mathematics will help them in their lives, not because they will see the same types of problems in the real world but because they are learning to think quantitatively and abstractly and developing an inquiry relationship with math.

Considerations for Implementation (Yeh et al., 2017):

I. **Physical and virtual environment.** Define spaces for individual, small-group, and whole-class work with easy access to learning tools and technology.
II. **Discourse-rich environment.** Facilitate the sharing of ideas at individual, small-group, and whole-class levels to support learners in being active and confident participants in mathematics.
III. **Task-rich environment.** Include worthwhile mathematics tasks in all aspects of teaching and assessment to establish a culture where learners expect regular math challenges and rise to meet them.
IV. **Assessment-rich environment.** Establish a norm where teachers and learners engage in ongoing assessment to inform both the teacher’s instructional decisions and learners’ next steps in learning. Mathematics assessment includes procedural and conceptual knowledge as well as the mathematics processes of problem solving, reasoning and proof, communication, connections, and representation.

Mathematics Identity

Mathematics identity includes beliefs about the nature of mathematics and engagement in mathematics, as well as beliefs about one's self as a mathematics learner, one's perceptions of how others perceive them as a mathematics learner, and perception of self as a potential participant in mathematics.

"Identity formation, including mathematics identity, is a lifelong process. This holds for the math teachers as well. As [teachers] focus more time and energy on supporting students in the development of their mathematics identity, [they] benefit from examining [their] own" (Allen & Schnell, 2016, p. 405).

Considerations for Implementation (Allen & Schnell, 2016):

I. **Know and believe in your students.** Get to know your students' history with, and beliefs about, mathematics. Identify and leverage each student's strengths in mathematics. Communicate and model the belief that all students can be successful in mathematics and that mathematics will help them in their lives.

II. **Redefine mathematical success.** Expertise in mathematical processes and problem solving strategies should be acknowledged and reinforced as much as correct answers. Speed, procedural fluency without conceptual understanding, or adherence to specified algorithms should not be criteria for student success or achievement.

III. **Prioritize student voice.** Create an environment where students are doing most of the work of reasoning and making sense of the mathematics. Support learners in metacognition, self- and peer-assessment, and goal-setting.

IV. **Monitor identity formation.** Use formative assessment not only to gather information on student understanding, but also on mathematics identity. Be aware of your own mathematical identity, and model the belief that the process of learning and doing mathematics is a life-long journey.

Routines

All mathematics classrooms should incorporate a variety of classroom routines used to activate prior learning, launch new instruction, and maintain fluency of important skills. Grade teams and school teams will collaboratively choose which routines are most appropriate for their context, and will plan for consistent use of these routines throughout the school year.

Classroom routines contribute to positive classroom culture by:

- making student thinking visible;
- maintaining and deepening prior skills and knowledge;
- incorporating games and play in mathematics; and
- providing ongoing assessment information for the teacher.

In an effective mathematics environment (Ontario Ministry of Education, 2020, p. 4), students know what it "looks like, sounds like, and feels like" to:

- work with a partner or small group;
- stick with a challenging problem;
- represent and communicate their thinking;
- use manipulatives;
- work independently;
- solve problems and reason logically;
- listen actively; and
- give and receive feedback.

Effective classroom routines include:

- energizers
- number talks
- data talks
- estimation routines
- thinking routines
- games

Mathematics Teaching Practices

The National Council of Teachers of Mathematics (NCTM) outlines eight practices that are essential to effective mathematics education. Each of these practices is examined and described in depth in *Taking Action: Implementing Effective Mathematics Teaching Practices* (Boston et al., 2017; Huinker & Bill, 2017; Smith et al., 2017). These books (at each grade-band) are the foundational texts for teacher development and professional learning in these practices.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

(NCTM, 2014, p. 10 – order of teaching practices altered)

Establish Mathematical Goals to Focus Learning

Mathematics instruction begins with the identification of learning goals that are focused on the mathematics that students will learn as a result of the lesson. These goals are learning intentions; statements that clearly describe what students will know, understand, and be able to do as a result of teaching and learning.

Learning goals that focus on understanding of mathematics, not just rote skills and procedures, "communicate the belief and expectation that all students are capable of participating and achieving in mathematics; in other words, such goals communicate a growth mindset" (Boston et al., 2017, p. 25).

Considerations for Implementation (Hattie et al., 2017; Huinker & Bill, 2017; Smith et al., 2017; Boston et al., 2017):

I. **Focus on understanding.** Write learning goals focused on what students will understand and connect those understandings to clear criteria for student learning.
II. **Connect to the Mathematics Curriculum.** Link learning goals to the Alberta Mathematics Curriculum or Program of Studies and to the CBE Assessment and Reporting Guides for Mathematics (where appropriate). Learning goals should encompass both the mathematical processes and grade-specific curriculum content.
III. **Situate learning goals within a learning progression.** Organize learning goals to build from lesson to lesson in a logical learning progression. To support an understanding of mathematics as an interconnected body of ideas, the learning goals should relate to essential understandings or big ideas of the discipline.
IV. **Use learning goals to guide planning.** Use learning goals to inform decisions about task design and selection, implementation strategies, and assessment.
V. **Share learning goals with students.** Foster student ownership of their learning by communicating learning goals and success criteria with students. "Research indicates significant gains in students' learning and reductions in achievement gaps when teachers communicate clear expectations, express challenging but attainable goals, and create an environment in which students feel supported to attain high goals" (Boston et al., 2017, p. 25).
VI. **Give feedback**. Provide students with timely, specific, and actionable feedback on their progress towards the learning goal.

Establish mathematical goals to focus learning Teacher and student actions	
What are *teachers* doing?	What are *students* doing?
Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit. Identifying how the goals fit within a mathematics learning progression. Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?) Using the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices. Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going. Assessing and monitoring their own understanding and progress toward the mathematics learning goals.

(NCTM, 2014, p. 16)

Implement Tasks that Promote Reasoning and Problem Solving

Tasks that promote reasoning and problem solving require high-level cognitive demand, have multiple entry and exit points, and can be solved using a variety of strategies. They are often "group-worthy," in that they give students something to work on collaboratively (Smith et al., 2017).

"A problem-solving activity must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Problem solving requires and builds depth of conceptual understanding and student engagement" (Alberta Education, 2007, updated 2016, p. 6).

Considerations for Implementation (Boston et al., 2017; Huinker & Bill, 2017; Smith et al., 2017):

I. **Emphasize high-level demand tasks.** Select and design tasks that provide higher-level demands (see chart below), have multiple entry and exit points, and multiple solution paths.
II. **Use related tasks.** Sequence related tasks over the learning progression to activate prior knowledge and build on student understanding.
III. **Provide ample time.** Give learners ample time to make sense of problems, explore possible solution paths, refine solutions and strategies, and reflect on their solutions and strategies.
IV. **Plan for scaffolds and extensions.** Provide parallel or modified versions of the task to support learners for whom additional support or challenge is needed.
V. **Maintain cognitive demand.** Be intentional in the teacher moves that maintain high cognitive demand for students throughout implementation of the task. Wait time, purposeful questions, and a culture of productive struggle all play a role in maintaining demand.
VI. **Teach about problem solving.** Include intentional instruction about the process of problem solving to make the required thinking explicit, and talk with students "about productive struggle, about making mistakes, and about adaptive reasoning" (Ontario Ministry of Education, 2020, p. 13).
VII. **Be inclusive.** Use a variety of culturally-relevant tasks to increase engagement and motivation of students.

Implement tasks that promote reasoning Teacher and student actions	
What are *teachers* doing?	What are *students* doing?
Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. Selecting tasks that provide multiple entry points through the use of varied tools and representations. Posing tasks on a regular basis that require a high level of cognitive demand. Supporting students in exploring tasks without taking over student thinking. Encouraging students to use varied approaches and strategies to make sense of and solve tasks.	Persevering in exploring and reasoning through tasks. Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas. Using tools and representations as needed to support their thinking and problem solving. Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

(NCTM, 2014, p. 24)

Task Criteria:

The criteria in the following chart can be used to analyze tasks and determine those that are high-level and provide opportunities for students to engage in reasoning and problem-solving.

Levels of Demand	
Lower-level demands (memorization) ▪ reproducing previously learned facts, rules, formulas, definitions or committing them to memory ▪ cannot be solved with a procedure ▪ have no connection to concepts or meaning that underlie the facts, rules, formulas, or definitions	Higher-level demands (procedures with connections) ▪ use procedure for deeper understanding of concepts ▪ broad procedures connected to ideas instead of narrow algorithms ▪ usually represented in different ways ▪ require some degree of cognitive effort; procedures may be used but not mindlessly
Lower-level demands (procedures without connections) ▪ are algorithmic ▪ require limited cognitive demand ▪ have no connection to the concepts or meaning that underlie the procedure ▪ focus on producing correct answers instead of understanding ▪ require no explanations	Higher-level demands (doing mathematics) ▪ require complex non-algorithmic thinking ▪ requires students to explore and understand the mathematics ▪ demand self-monitoring of one's cognitive process ▪ require considerable cognitive effort and may involve some level of anxiety because solution path isn't clear

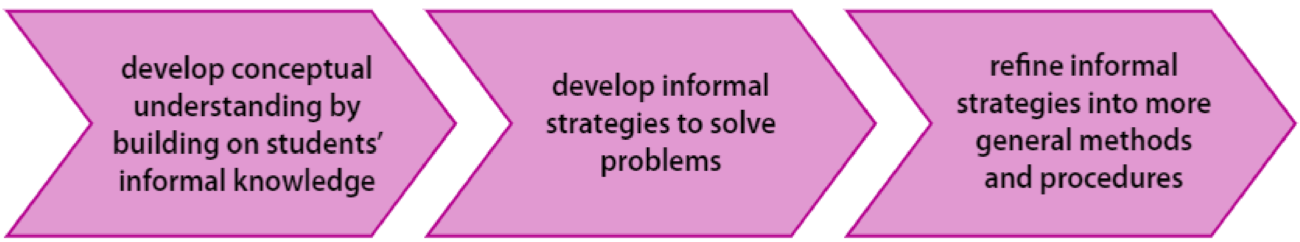
(Adapted from Boston et al., 2017, pp. 32-33; Huinker & Bill, 2017, pp. 41-42; Smith et al., 2017, pp. 32-33)

Build Procedural Fluency from Conceptual Understanding

Conceptual understanding refers to "the comprehension and connection of concepts, operations, and relations" (NCTM, 2014, p. 7). It is the ability to explain how mathematical operations or procedures relate to a physical context or process, how mathematical rules or procedures are derived, and how operations and procedures relate to each other. Conceptual understanding "establishes the foundation, and is necessary, for developing procedural fluency" (NCTM, 2014, p. 7).

Procedural fluency refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. "Fluency builds from initial exploration and discussion of number concepts to using informal reasoning strategies based on meanings and properties of the operations to the eventual use of general methods as tools in solving problems" (NCTM, 2014, p. 42).

Tasks are purposefully sequenced so students:



(Huinker & Bill, 2017, p. 90)

Considerations for Implementation (Boston et al., 2017; Huinker & Bill, 2017; Smith et al., 2017):

I. **Use a variety of representations.** Model and use a variety of concrete, visual, and symbolic representations, consistently making connections between the representations.

II. **Focus on conceptual understanding.** Recognize that students that have conceptual understanding in connection with procedures are more likely to see mathematics as making sense, remember the procedures, and apply procedures in appropriate ways in new situations.

III. **Encourage multiple strategies.** Model and foster the use of multiple strategies to solve problems or use procedures. Conceptual understanding allows students to use procedures in a flexible way and develop confidence and agency in using mathematics. Students who have a variety of procedures to choose from will have greater number sense and flexibility.

IV. **Use classroom routines.** Use routines such as number talks, estimation routines, and games to support learners in developing mental math skills and automaticity.

V. **Include problem solving.** Include contextual and mathematical problems at all points in the learning progression to reinforce conceptual understanding and give learners opportunities to choose appropriate and efficient procedures.

VI. **Provide deliberate practice**. Implement a variety of forms of practice that includes all aspects of student learning (i.e., conceptual understanding, procedures, representation, communication, and problem solving). Practice should be purposeful and spaced.

Build procedural fluency from conceptual understanding Teacher and student actions	
What are *teachers* doing?	What are *students* doing?
Providing students with opportunities to use their own reasoning strategies and methods for solving problems. Asking students to discuss and explain why the procedures that they are using work to solve particular problems. Connecting student-generated strategies and methods to more efficient procedures as appropriate. Using visual models to support students' understanding of general methods. Providing students with opportunities for distributed practice of procedures.	Making sure that they understand and can explain the mathematical basis for the procedures that they are using. Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems. Determining whether specific approaches generalize to a broad class of problems. Striving to use procedures appropriately and efficiently.

(NCTM, 2014, p. 47-48)

Pose Purposeful Questions

Purposeful questions are part of instructional design, prompting students to make connections and explore personal strategies and solution paths. In addition, questioning is a powerful tool in maintaining a high level of cognitive demand in tasks and supporting productive struggle.

The mathematical ideas students are exposed to, as well as their mathematical identities, are impacted by the types of questions teachers ask, the way in which they ask questions, who they ask questions of, and the way they hold students accountable for answering all questions (Boston et al., 2017; Huinker & Bill, 2017; Smith et al., 2017).

Considerations for Implementation (Boston et al., 2017; Huinker & Bill, 2017; Smith et al., 2017):

I. **Use a balance of question types.** Ask different types of questions depending upon the purpose of the question. Different types of questions cue learners to think and respond in different ways such as, recalling facts, explaining their thinking, making connections, reflecting on their work, or engaging with others' ideas.
II. **Use wait time.** Allow for adequate wait time after asking a question and after students respond to allow for thinking and to provide opportunity for all students to engage.
III. **Plan key questions.** Anticipate and pre-plan questions to be ready to respond to students' strategies, representations, and misconceptions without taking over the thinking for them.
IV. **Develop students' ability to respond effectively.** Provide students with sentence stems to develop their ability to communicate mathematically and to recognize when others are revoicing, adding on, agreeing or disagreeing, asking questions, or rethinking.
V. **Reflect on questioning.** Take time to reflect on the effectiveness of questions asked and analyze the question patterning in the classroom to ensure equity for all learners.

Pose purposeful questions Teacher and student actions	
What are *teachers* doing?	What are *students* doing?
Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. Allowing sufficient wait time so that more students can formulate and offer responses.	Expecting to be asked to explain, clarify, and elaborate on their thinking. Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly. Reflecting on and justifying their reasoning, not simply providing answers. Listening to, commenting on, and questioning the contributions of their classmates.

(NCTM, 2014, p. 41)

Questions That Foster Discussion:

I. **Helping students work together to make sense of mathematics**
- What do others think about what ____said?
- Do you agree? Disagree?
- Does anyone have the same answer but a different way to explain it?
- Can you convince the rest of us that that makes sense?

II. **Helping students rely more on themselves to determine whether something is mathematically correct –**
- Why do you think that?
- Why is that true?
- How did you reach that conclusion?
- Does that make sense?
- Can you design a model to show that?

III. **Helping students learn to reason mathematically –**
- Does that always work?
- Is that true for all cases?
- Can you think of a counterexample?
- How could you prove that?
- What assumptions are you making?

IV. **Helping students to learn to conjecture, invent, and solve problems –**
- What would happen if…? What if not?
- Do you see a pattern?
- What are some possibilities here?
- Can you predict the next one? What about the last one?
- How did you think about the problem?
- What decision do you think he should make?
- What is alike and what is different about your method of solution and hers?

V. **Helping students connect mathematics, its ideas and applications –**
- How does this relate to . . .?
- What ideas that we have learned were useful in solving this problem?
- Have we ever solved a problem like this before?
- Can you give me an example of…?

(Boston et al., 2017, p. 77-78; Smith et al., 2017, p. 83)

Use and Connect Mathematical Representations

Representations provide access to abstract mathematical ideas (National Research Council, 2001). Mastery of mathematics outcomes requires flexibility with various representations and the strategic choice of different representations for different purposes. Connections between representations should be made explicit at all stages of learning to support and maintain conceptual understanding and procedural fluency.

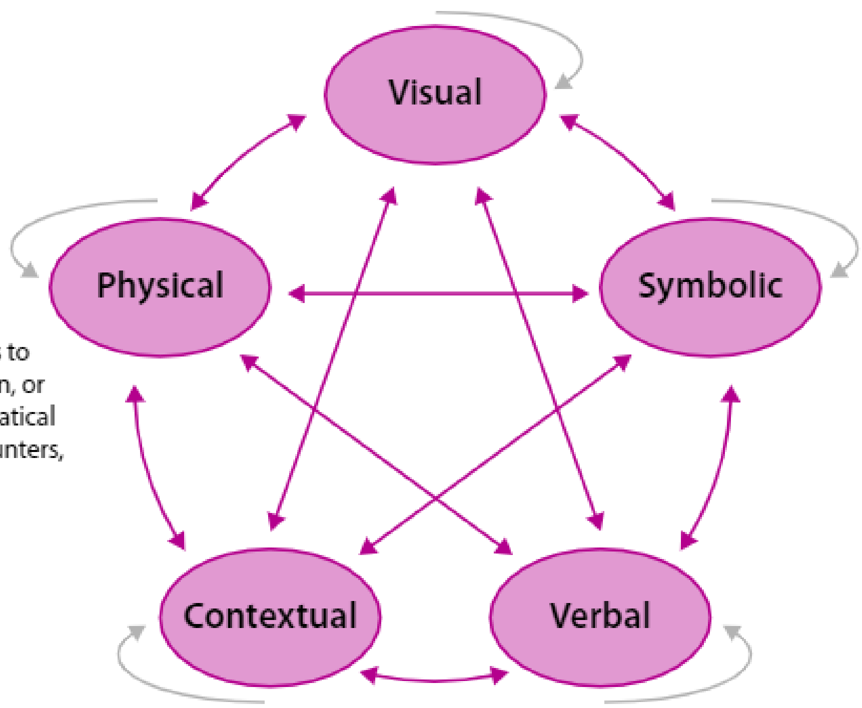
Focusing on multiple forms of representation is important for equity and access for all students. Using cultural contexts familiar to students acknowledges their cultural experiences and recognizes the validity of mathematics strategies from around the world (Boston et al., 2017; Huinker & Bill, 2017; Smith et al., 2017).

Visual

Illustrate, show, or work with mathematical ideas using diagrams, pictures, number lines, graphs, and other math drawings.

Physical

Use concrete objects to show, study, act upon, or manipulate mathematical ideas (e.g. cubes, counters, tiles, paper strips)



Symbolic

Record or work with ideas using numerals, variables, tables, and other symbols.

Contextual

Situate mathematical ideas in everyday, real-world, imaginary, or mathematical situations and contexts.

Verbal

Use language (words and phrases) or interpret, discuss, define, or describe mathematical ideas, bridging informal and formal mathematical language.

(Definitions: Huinker, 2015, p. 5)
(Image: Huinker & Bill, 2017, p. 137)

Considerations for Implementation (Boston et al., 2017; Huinker & Bill, 2017; Smith et al., 2017):

I. **Use direct instruction.** Provide instruction for all five types of representations to focus students' attention on the structure and essential features of the representations and mathematical ideas.
II. **Make connections explicit.** Make connections between and within all types of representations in alternating directions. Students should compare and contrast representations, identifying strengths and weaknesses of different representations in relation to the problem or mathematical idea.
III. **Integrate with mathematical discourse.** Encourage learners to use representations as tools to empower them to make sense of mathematical ideas, solve problems, and communicate reasoning.
IV. **Model and develop flexibility.** Have students explain decisions about which representations to use in solving problems or communicating thinking.

Use and connect mathematical representations Teacher and student actions	
What might *teachers* be doing?	What might *students* be doing?
Selecting tasks that allow students to decide which representations to use in making sense of the problems. Allocating substantial instructional time for students to use, discuss, and make connections among representations. Introducing forms of representations that can be useful to students. Asking students to make math drawings or use other visual supports to explain and justify their reasoning. Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	Using multiple forms of representations to make sense of and understand mathematics. Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations. Making choices about which forms of representations to use as tools for solving problems. Sketching diagrams to make sense of problem situations. Contextualizing mathematical ideas by connecting them to real-world situations. Considering the advantages or suitability of using various representations when solving problems.

(NCTM, 2014, p. 41)

Facilitate Meaningful Mathematical Discourse

Mathematics discourse occurs when students are engaged in communication about their mathematical thinking and reasoning in verbal, written, or visual forms. Through rich discussion, student exchange ideas, agree, disagree, conjecture, and justify their thinking as they make sense of the mathematics together (NCTM, 2014).

Considerations for Implementation (Boston et al., 2017; Huinker & Bill, 2017; Smith et al., 2017):

I. **Use various modes of communication.** Model and use verbal, written, or visual forms of communication.
II. **Promote problem solving and reasoning.** Select and implement tasks that provide opportunities for learners to engage in discussion and questioning.
III. **Develop mathematical language.** Model consistently and explicitly the use of mathematical language using specific vocabulary acquisition techniques such as word walls and concept mapping. Mathematical discourse plays a critical role in the development of mathematical vocabulary. Meaningful classroom discussions provide students opportunities to use the new vocabulary to make sense of their thinking.
IV. **Cultivate a positive mathematics environment.** Create a positive and rich mathematics culture by considering the physical and emotional environment. To support learners in sharing their own thinking and responding to others' thinking, establish norms and routines such as the use of Talk Moves.
V. **Pre-plan and organize discussions**. Use advance preparation to make mathematical discussions more manageable and reduce the number of in-the-moment decisions required. Teachers can anticipate how students are likely to approach problems and design their responses to those solution strategies in advance.

Facilitate meaningful mathematical discourse Teacher and student actions	
What are *teachers* doing?	What are *students* doing?
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.	Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.
Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.	Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.
Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.	Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others.
Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Identifying how different approaches to solving a task are the same and how they are different.

(NCTM, 2014, p. 35)

Elicit and Use Evidence of Student Thinking

The progressive and cumulative nature of mathematics requires teachers to be constantly gathering information about where students are in their factual, procedural, and conceptual knowledge. Teachers need to engage with students' existing knowledge so that teaching and learning can build on current understandings and address underlying misconceptions that are barriers to further learning.

"Considerable enhancements in student achievement are possible when teachers use assessment, minute-by-minute and day-by-day, to adjust their instruction to meet their students learning needs" (Wiliam, 2007, p. 4).

Considerations for Implementation (Boston et al., 2017; Huinker & Bill, 2017; Smith et al., 2017):

I. **Collect a balanced body of evidence.** Listen to students' words, observe their actions, or review their written work, including pictures, words, and symbols.
II. **Use mathematics discourse.** Ask students to explain and justify their response to a question or task to gain insight into how students are reasoning and how they are choosing strategies and procedures.
III. **Use tasks that promote reasoning and problem solving.** Ensure that tasks provide sufficient entry and exit points for all learners to demonstrate where they are in their learning and allow for flexibility in strategies and representations used.
IV. **Foster student ownership of learning.** Support students in being active participants in the learning process as they ask questions, monitor their progress towards learning goals, and articulate their next steps for growth.
V. **Adjust instruction.** Respond to evidence by adjusting planning, instruction, and tasks according to evidence. Adjustments are made as part of in-the-moment instructional decisions as well as in short and long-term planning and instruction.
VI. **Calibrate to CBE standards.** Use the Assessment and Reporting Guides for Mathematics to plan for assessment and next steps in learning.

Elicit and use evidence of student thinking Teacher and student actions	
What are *teachers* doing?	What are *students* doing?
Identifying what counts as evidence of student progress toward mathematical goals. Eliciting and gathering evidence of student understanding at strategic points during instruction. Interpreting student thinking to assess mathematical understanding, reasoning, and methods. Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend. Reflecting on evidence of student learning to inform the planning of next instructional steps.	Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse. Reflecting on mistakes and misconceptions to improve their mathematical understanding. Asking questions, responding to, and giving suggestions to support the learning of their classmates. Assessing and monitoring their own progress towards mathematics learning goals and identifying areas in which they need to improve.

(NCTM, 2014, p. 56)

Support Productive Struggle in Learning Mathematics

Struggle in mathematics occurs when students face a challenging task that they do not immediately know how to solve. Students are engaging in productive struggle when they engage in tasks that require them to wrestle with critical mathematical ideas that are "within reach but present enough challenge" (Hiebert & Grouws, 2007, p. 388).

Productive struggle gives students the opportunity to develop positive mathematics identities as they are encouraged to see struggle as an essential part of learning, and themselves as competent problem solvers.

Considerations for implementation (Boston et al., 2017; Huinker & Bill, 2017; Smith et al., 2017):

I. **Redefine student and teacher success**. Recognize that observing students struggle mathematically can be challenging, as all teachers want their students to succeed. If teachers perceive the struggle as an indicator of unsuccessful teaching, they may reduce or remove the struggle by breaking the problem down and guiding students' systematically toward a solution. Although done with good intention, it "undermines the efforts of students, lowers the cognitive demand of the task, and deprives students of opportunities to engage fully in making sense of the mathematics" (NCTM, 2014, p. 48).

II. **Give adequate time**. Provide ample time for learners to explore, reason, conjecture, and revise as they wrestle with emerging mathematical ideas, and provide time for individual and group reflection during problem solving activities.

III. **Ask purposeful questions**. Keep the struggle in a productive zone by asking students to explain how they solved a problem, justify why a particular strategy works, or explain another approach to solving the same problem. This may surface student errors or misconceptions that the teacher can build upon, thus leading to new insights and allowing the student to move forward in their thinking.

IV. **Acknowledge student contributions**. Recognize students' thinking contributions without taking the thinking away from the students. Encourage students to listen and make sense of others' reasoning.

V. **Develop metacognition.** Support learners in the process of reflecting on *what they know*, *how they know it* and *what they need to know next* to make sense of the problem at hand. Students learn to take ownership of their mathematical understanding by developing metacognition skills.

VI. **Provide encouragement**. Recognize effort and promote openness to mistakes as a natural part of learning mathematics.

Support productive struggle in learning mathematics Teacher and student actions	
What are *teachers* doing?	What are *students* doing?
Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle.	Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle.
Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them.	Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks.
Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles.	Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up.
Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	Helping one another without telling their classmates what the answer is or how to solve the problem.

(NCTM, 2014, p. 52)

Mathematics Assessment Practices

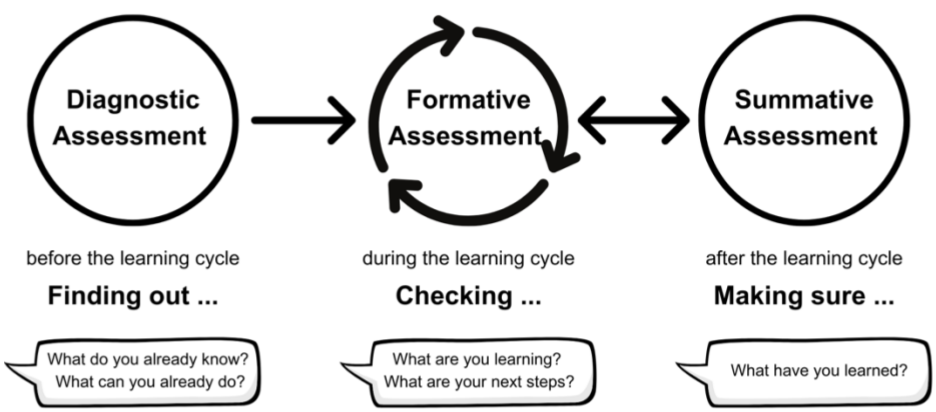
"It is only through assessment that we can discover whether the instructional activities in which we engaged our students resulted in the intended learning. Assessment really is the bridge between teaching and learning" (Wiliam, 2013, p.15).

Assessment and Reporting in CBE

CBE's foundational assessment guidelines are outlined in Assessment and Reporting in CBE. These guidelines are based on the premise that the primary purpose of assessment is to improve student learning and are central to all assessment and reporting policies and practices. The five guiding principles in CBE are:

- Assessment practices are fair, transparent, and equitable for all students.
- Assessment makes explicit connections to the intended learning goals.
- Assessment is ongoing and embedded throughout cycles of learning.
- Students are actively involved in the assessment process.
- Assessment information shared with students and families is clear and meaningful.

Assessment types throughout a learning cycle.



Diagnostic Assessment

Teachers collect diagnostic assessment information at the beginning of each learning cycle to determine what students already know, understand and can do in relation to the learning goals. Diagnostic assessments may be administered to individual students, small groups and/or the whole class. As these are low-stake assessments, the results are not used in the determination of report card grades or to make recommendations for course enrolment.

A broad range of diagnostic assessments can be used to provide teachers with valuable information which:

- informs planning and instruction;
- identifies learner interests, learning preferences, current level of understanding and/or readiness to learn new skills;
- supports differentiation and scaffolding of learning for students; and
- identifies students that may require further supports, targeted interventions and/or more specific diagnostic information gathering.

Teachers attend to equity practices and students' mathematical identities throughout the implementation of commercially prepared or teacher-designed diagnostic assessments. When schools are administering diagnostic assessments, please consider:

- attending to the validity, alignment, reliability and potential bias of assessments;
- providing required supports; and
- mitigating the emotional impact.

Formative Assessment

The majority of classroom assessment is formative. It is through ongoing assessments that we assess the impact of the teaching and learning activities and allow for responsive instructional decision making that supports successful student achievement of the intended learning goals. In most cases, formative assessment does not and should not be used when determining report card grades.

Wiliam and Thompson (2005) outline five strategies that support effective formative assessment:

1 | Clarifying, sharing and understanding learning intentions and success criteria
2 | Engineering effective discussion, tasks and activities that elicit evidence of learning
3 | Providing feedback that moves learners forward
4 | Activating students as learning resources for each other
5 | Activating students as owners of their own learning

Implementation of the eight mathematics teaching practices facilitates formative assessment, giving teachers a window into the reasoning behind student responses and an opportunity to respond and target instruction and personalize learning for students.

Summative Assessment

Summative assessment occurs at or near the end of each learning cycle, after multiple opportunities for formative feedback have been provided and teachers are reasonably confident that learning has taken place and students are ready to have their learning evaluated. These assessments measure achievement of and progress towards the intended learning goals.

By designing summative assessment that employ research-based best practices, as described in the *High Quality Summative Assessment* resources (i.e., K-9, 10-12, Modified Programming), teachers ensure report card information is as accurate and fair as possible. This is further supported through triangulation of assessment evidence in regard to mode (observation, conversation, product) and frequency. Summative assessment information informs final grade determination as well as report card comments.

Common Assessments

The use of common assessments for diagnostic, formative or summative purposes provides consistency and facilitates ongoing adjustments to planning and Professional Learning Community (PLC) work. These assessments are developed and administered in a given timeframe by teachers in a grade level / course or across grade levels / courses in a particular subject area to determine if students are mastering the intended learning outcomes. Teachers analyze data collaboratively and plan next steps to ensure a continual focus on improving student learning. Designing and administering common assessments are a school-based decision.

References

The following resources are referenced in this document. See the full list of documents referred to in the writing of the framework in *K – 12 Mathematics Framework | History and Context*.

Alberta Assessment Consortium. (2021). https://aac.ab.ca/

Alberta Education. (2022). Implementing a Continuum of Supports and Services: A Resource Guide for School and School Authority Leaders. Edmonton, AB; Alberta Education.

Alberta Education. (2007, updated 2016). *Mathematics Kindergarten to Grade 9 [Program of studies]*. https://education.alberta.ca/media/3115252/2016_k_to_9_math_pos.pdf

Alberta Education. (2008). *Mathematics Grades 10-12 [Program of studies]*. https://education.alberta.ca/media/564028/math10to12.pdf

Alberta Education. (n.d.). *Learn Alberta Planning Guides*. Learn Alberta. https://www.learnalberta.ca/Search.aspx?lang=en&search=planning+guide&grade

Alberta Education. (2020-2021). *General Information Bulletin Grades 6 and 9: Alberta Provincial Achievement Testing 2020-2021.* Alberta Education. https://www.alberta.ca/assets/documents/edc-pat-general-information-bulletin.pdf

Alberta Education. (2021). *Diploma Exams – Overview*. Alberta Education. https://www.alberta.ca/diploma-exams-overview.aspx#jumplinks-2

Allen, K. & Schnell, K. (2016). Developing mathematics identity. *Mathematics Teaching in the Middle School, 21*(7), 398-405. https://www.asdn.org/wp-content/uploads/Developing-Math-Identity-1.pdf

Berry III, R. Q. (2019, May). *Examining Equitable Teaching Using the Mathematics Teaching Framework*. NCTM President's Message. https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Robert-Q_-Berry-III/Examining-Equitable-Teaching-Using-the-Mathematics-Teaching-Framework/

Boaler, J. (n.d.). When you believe in your students they do better. youcubed at Stanford University. Retrieved March 1, 2021, from https://www.youcubed.org/wp- content/uploads/2017/05/When-You-Believe-in-Your-Students-They-Do-Better.pdf

Boston, M., Dillon, F., Smith, M. S., & Miller, S. (2017). *Taking action: Implementing effective mathematics teaching practices in Grades 9-12*. National Council of Teachers of Mathematics.

Calgary Board of Education. *Assessment and Reporting in CBE*.

Calgary Board of Education. (n.d.). *CBE Education Plan | 2021-2024.* https://www.cbe.ab.ca/FormsManuals/Education-Plan.pdf

Edmonton Public Schools. (2020). *Math Intervention/Programming Instrument (MIPI)*. Edmonton Public Schools.

Eichhorn, M. S., DiMauro, P. J., Lacson, C., & Dennie, B. (2019). Building the optimal learning environment for mathematics. *Mathematics Teacher, 112*(4), 263-267. https://doi.org/10.5951/mathteacher.112.4.0262

Hattie, J., Fisher, D., & Frey, N., Gojak, L. M., Moore, S. D., & Mellman, W. (2017). *Visible learning for mathematics, grades K-12: What works best to optimize student learning*. Corwin Mathematics.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Retrieved from: http://citeseerx.ist.psu.edu/viewdoc/download?_doi=10.1.1.405.3591&rep=rep1&type=pdf

Huinker, D. (2015). Representational competence: A renewed focus for classroom practice in mathematics. *Wisconsin Teacher of Mathematics*, *67*(2), 4-8. https://doi.org/10.1007/978-1-4419-7582-9_5

Huinker, D., & Bill, V. (2017). *Taking action: Implementing effective mathematics teaching practices in K-Grade 5*. National Council of Teachers of Mathematics.

National Council of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. National Council of Mathematics.

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: Ensuring mathematical success for all*. National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). (2020a). *Catalyzing change in early childhood and elementary mathematics: Initiating critical conversations.* National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). (2020b). *Catalyzing change in middle school mathematics: Initiating critical conversations.* National Council of Teachers of Mathematics.

National Research Council, & Mathematics Learning Study Committee. (2001). *Adding it up: Helping children learn mathematics*. National Academies Press

Ontario Ministry of Education. (2020. *High-impact instructional practices in mathematics*. Ontario Public Services. Retrieved from http://www.edu.gov.on.ca/eng/teachers/high-impact-instruction-math.pdf

Smith, M. S., Steele, M. D., & Raith, M. L. (2017). *Taking action: Implementing effective Mathematics teaching practices in Grades 6-8*. National Council of Teachers of Mathematics.

Wiliam, D. (2007). Five "key strategies" for effective formative assessment. Retrieved from https://www.nctm.org/Research-and-Advocacy/Research-Brief-and-Clips/Strategies-for-Formative-Assessment/

Wiliam, D. (2013). Assessment: The bridge between teaching and learning. *Voices from the Middle*, *21*(2), 15-20.

Yeh, C., Ellis, M. W., & Hurtado, C. K. (2017). *Reimagining the mathematics classroom: Creating and sustaining productive learning environments, K-6*. The National Council of Teachers of Mathematics.